Analytical approach to investigate exospheric density profiles and escape flux due to the radiation pressure from the Sun

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Outline

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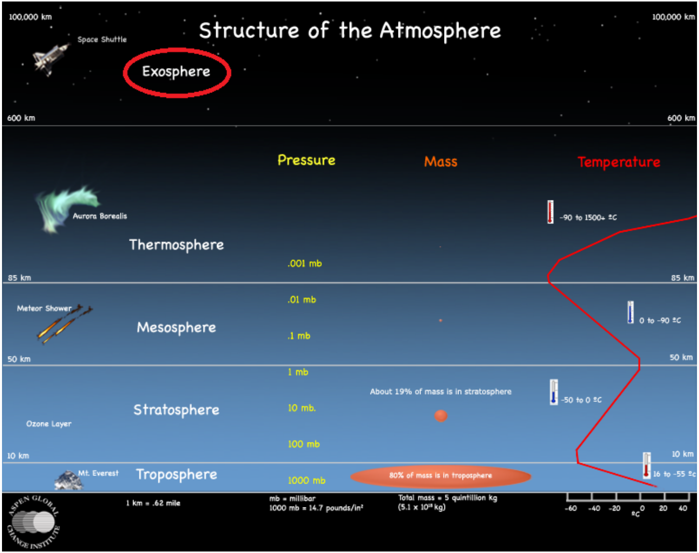
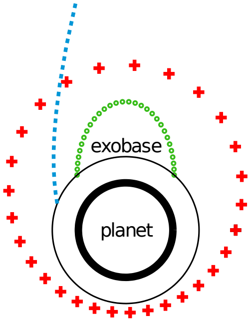
1. Description of the exosphere
2. Modeling
   - Chamberlain (1963) approach
   - Bishop and Chamberlain (1989) approach
   - Generalization of Bishop and Chamberlain to 3D case
3. Results
   - Densities
   - Thermal flux
4. Conclusions

Description



Exospheric populations

The exosphere is a quasi collisionless medium. The trajectories of the particles depend essentially on gravity (and radiation pressure at higher distances). In theory, there are five kinds of particles.



- ballistic : their trajectory describes an ellipse but their periapsis is below the exobase
- satellite : their trajectory is also an ellipse but their periapsis is above the exobase
- escaping : their speed is higher than the escape velocity
- hyperbolic : coming from infinity and passing by (neglected)
- hyperbolic : coming from infinity and undergoing collisions with lower layers of the atmosphere (neglected)

Outline

Chamberlain (1963) approximation

Chamberlain (1963) proposed an approximation for the satellite populations which is a priori overestimated. With this approximation, one can show that the satellite particles could be even dominant in the exospheres at large altitudes in particular for light species with small λ.

$$n(r) = n_{bar}\zeta(\lambda) = n(r_{exo})e^{\lambda-\lambda_{exo}}(\zeta_{bal} + \zeta_{esc} + \zeta_{sat})$$

$$\lambda(r) = \frac{GMm}{k_B T_{exo} r}$$

Bishop and Chamberlain (1989) approach

Study the Hamiltonian of a particle subject to the gravitational potential of the Earth and a constant force coming from the Sun, the radiation pressure, inducing a constant acceleration f.

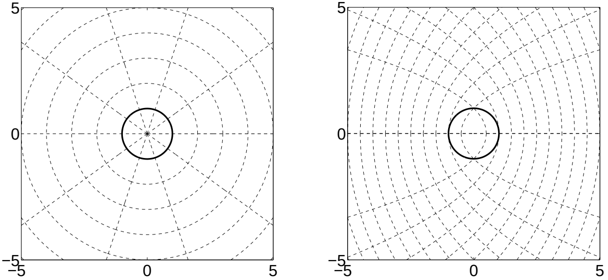
$$\vec{a} = -\frac{GM}{r^2}\vec{e}_r - f\vec{e}_x$$

The X-axis points from Earth to the Sun. This problem is analogous to the Stark effect. Commonly, to solve this problem, we change the system of coordinates.

$$R_{lim} = \sqrt{\frac{GM}{f}}$$

$$\lambda_a = \lambda(R_{lim})$$

Bishop and Chamberlain (1989) approach



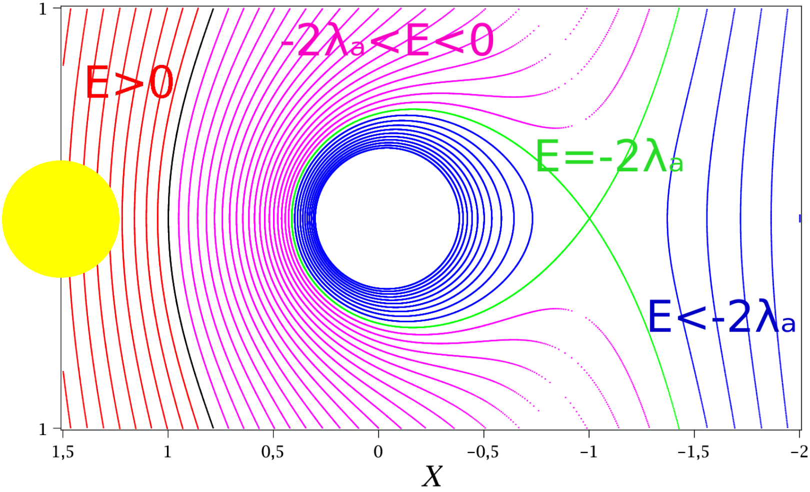
If θ is the angle between $\vec{e}_x$ and $\vec{e}_r$. The new system of coordinates is :

$$u = r + x = r(1 + \cos\theta)$$

$$w = r - x = r(1 - \cos\theta)$$

We keep the angle ϕ of rotation around the Sun-planet axis

Equipotential lines



Theory or not ?

The theory is based on hamiltonian mechanics and the treatment is similar to the so-called Stark effect. I will spare you this except if you will be part of my Ph. D. thesis committee ...

Restriction for the motion in 3D

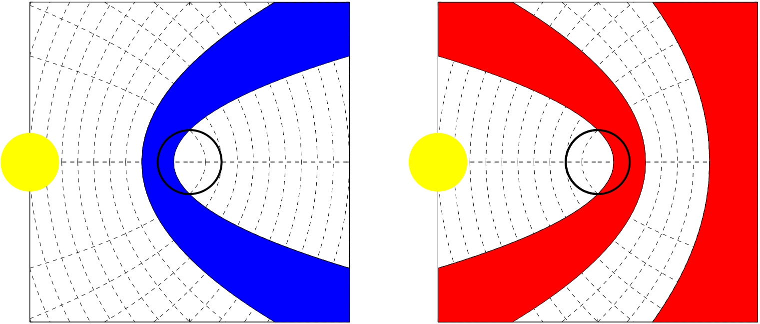


FIGURE : Restrictions for the motion in space. Left panel : restriction for u-motion. Right panel : restriction for w-motion.

Restriction for the motion in 3D

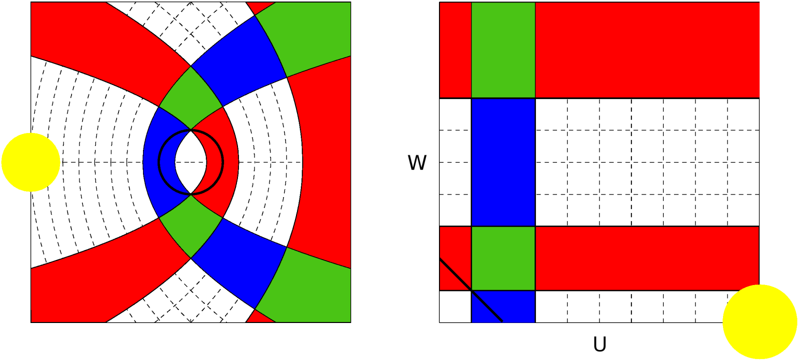


FIGURE : Combination of two previous restrictions in the common plan and in the (u,w) frame.

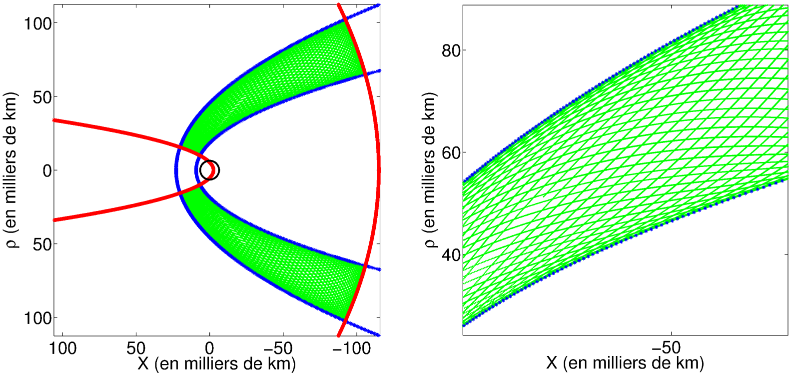
Restriction for the motion in 3D

Two kinds of motion :

- Bounded : the closed green rectangle. The motion is constrained in this area and the particle moves in this whole space.
- Unbounded : the open green rectangle. The particle is not linked to the planet and escapes.

So, the particles are ballistic or escaping if the green regions cross the exobase.

Example of a simulated trajectory for a bounded particle



What for ?

To determine densities and escaping flux semi-analytically

Supposing a collisionless exosphere $\Longrightarrow$ Liouville theorem

$\Longrightarrow$ the velocity distribution function (VDF) is a TRUNCATED maxwellian distribution because we need the exospheric particles come from the exobase.

All velocities, depending on positions in space, are not possible. We have constraints on initial conditions of the particle to distinguish its type : ballistic, escaping or satellite. At a given position, we can know for the integration if the velocities that we take are possible or not for the particle to cross the exobase.

Partition function

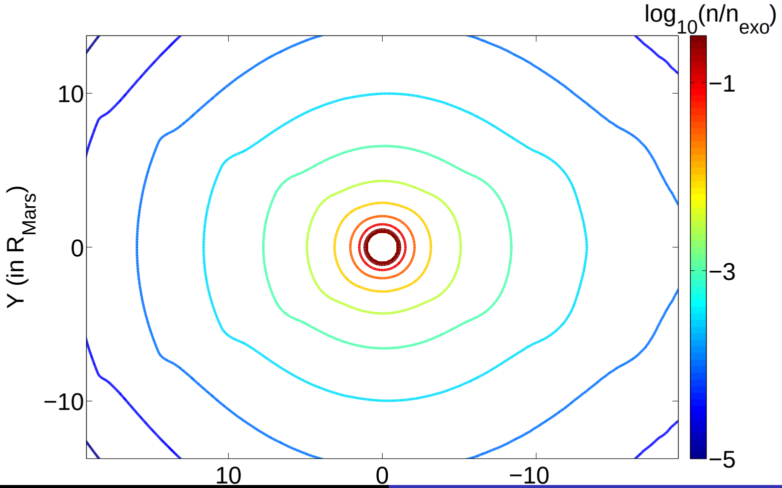
$$\frac{n(u,w)}{n_{exo}} = \underbrace{\exp(\lambda-\lambda_c)}_{\text{barometric law}} \underbrace{\exp\left(-\frac{\lambda_a(u-w)}{2R_{pressure}}\right)}_{\text{radiation pressure}} \zeta(u,w)$$

$$\lambda_a = \lambda(R_{pressure}) = \frac{GMm}{k_B T_{exo} R_{lim}} = \frac{\sqrt{GMf}\, m}{k_B T_{exo}}$$

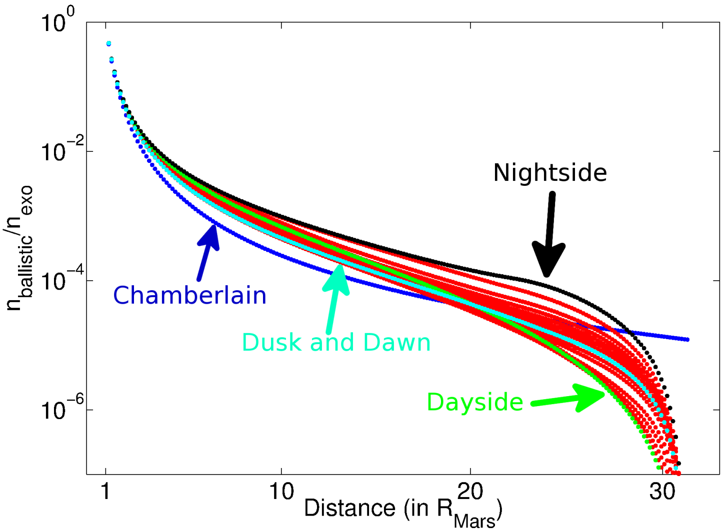
$$\zeta_{\text{type}}(u,w) = \frac{\displaystyle\int_{\text{type}} \exp\left(-\frac{p^2}{2mk_B T}\right) \mathrm{d}^3\vec{p}}{\displaystyle\int \exp\left(-\frac{p^2}{2mk_B T}\right) \mathrm{d}^3\vec{p}} = \frac{\displaystyle\int \mathbb{1}_{\text{type}} \exp\left(-\frac{p^2}{2mk_B T}\right) \mathrm{d}^3\vec{p}}{\displaystyle\int \exp\left(-\frac{p^2}{2mk_B T}\right) \mathrm{d}^3\vec{p}}$$

Principle : at a given position, in all velocity phase space, for each point of integration, we check if the combination of the initial conditions and velocities allow to the particle to cross the exobase or not, once or twice.

Ballistic particles density for H on Mars including radiation pressure



Ballistic particles density : comparison with Chamberlain

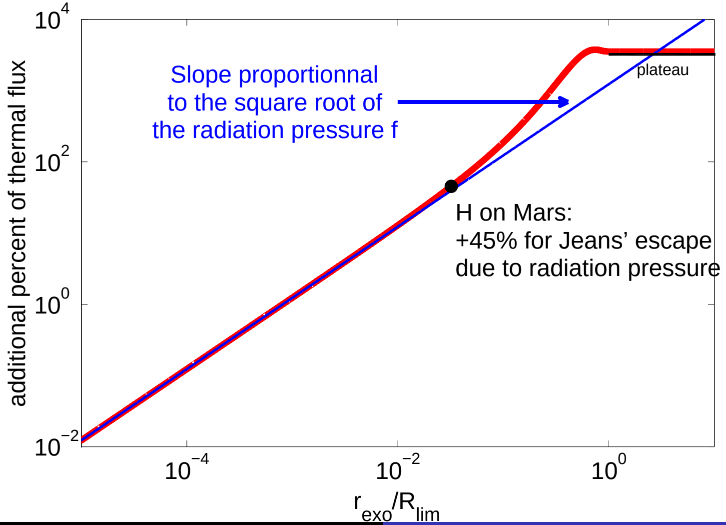


Ballistic particles density : comparison with Chamberlain

- The densities are increased by up to a factor of 4, compared with Chamberlain's profiles that do not include the radiation pressure influence
- The exosphere has a distance limit, an exopause, due to the radiation pressure at the same distance in all directions (not proved analytically, I try)

Analytical escaping flux : relative error to Jeans' flux

Escaping flux at noon in addition to Jeans' escape in percent for H on Mars



This semi-analytical approach allows to determine exactly the density profiles of each kind of particles, in theory. In practice, now, only the ballistic particles density is easy to evaluate and the flux at noon or midnight.

In the future, we hope to extend the calculation of the flux to all the exobase and the density of escaping particles to the whole exosphere.